QP CODE: 26018593

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Reg No : .....

Name : .....

MSc DEGREE (CSS) SPECIAL REAPPEARANCE EXAMINATION, MARCH 2026

Third Semester

M.Sc. MATHEMATICS

CORE COURSE - ME010302 - PARTIAL DIFFERENTIAL EQUATIONS

2024 ADMISSION ONLY

5D2118FB

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the integral curves of $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$
2. Eliminate the arbitrary function f from the equation $z = f(\frac{xy}{z})$
3. Find the general integral of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$.
4. What do you mean by complete integral of a nonlinear partial differential equation? How will you derive the general integral and singular integral from a complete integral?
5. Fina a complete integral of the equation $p + q = pq$.
6. Find the complementary function of $25r - 40s + 16t = 0$.
7. Find the particular integral of $[D^2 - D']z = e^{x+y}$.
8. Classify the equation $u_{xx} + x^2 u_{yy} = 0$.
9. Establish a formula for finding the potential function of a family of equipotential surfaces.
10. Prove that the function $\phi = x + \frac{x}{x^2+y^2}$ is Harmonic.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the equations of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersections with the hyperboloids of the one-parameter system $xy = z + c$.

12. Verify that the equation $yz\,dx + xz\,dy + xy\,dz = 0$ is integrable and if so find its primitive.

13. Find the complete integral of the equation $p = (z + qy)^2$.

14. Find the complete integral of the equation $2(y + zq) = q(xp + yq)$.

15. Verify that the PDE $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ is satisfied by $z = \frac{1}{x}\phi(y - x) + \phi'(y - x)$.

16. Show that the equation $\frac{\partial^2 y}{\partial t^2} + 2k\frac{\partial y}{\partial t} = c^2\frac{\partial^2 y}{\partial x^2}$ possesses solutions of the form $\sum_{r=0}^{\infty} C_r e^{-kt}\cos(\alpha_r x + \varepsilon_r).\cos(w_r t + \delta_r)$ where $C_r, \alpha_r, \varepsilon_r, \delta_r$ are constants and $w_r^2 = \alpha_r^2 c^2 - k^2$.

17. By separating the variables solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k}\frac{\partial z}{\partial t}$.

18. Prove that $rcos\theta$ and $r^{-2}cos\theta$ satisfy the Laplace's equation , when r, θ, ϕ are spherical polar coordinates.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Prove the following.
a) If X is a vector such that $X \cdot curlX = 0$ and μ is an arbitrary function of $x, y, z,$ then $(\mu X) \cdot curl(\mu X) = 0$.
b) A necessary and sufficient condition that the Pfaffian differential equation $X \cdot r = 0$ should be integrable is that $X \cdot curlX = 0$.

20. a) If $u_i(x_1, x_2, \ldots, x_n, z) = c_i, (i = 1, 2, \ldots, n)$ are the independent solutions of the equations $\frac{dx_1}{P_1} = \frac{dx_2}{P_2} = \ldots = \frac{dx_n}{P_n} = \frac{dz}{R}$, then prove that the relation $\Phi(u_1, u_2, \ldots, u_n) = 0$, in which Φ is arbitrary, ia a general solution of the linear partial differential equation $P_1\frac{\partial z}{\partial x_1} + P_1\frac{\partial z}{\partial x_2} + \ldots + P_1\frac{\partial z}{\partial x_n} = R$.
b) Find the general solution of the partial differential equation $(y - z)\frac{\partial u}{\partial x} + (z - x)\frac{\partial u}{\partial y} + (x - y)\frac{\partial u}{\partial z} = 0$ if u is a function of x, y and z.

21. By Jacobi's method, solve $z^2 + zu_z - p^2 - q^2 = 0$.

22. (a) State and prove the necessary condition that a family of surfaces $f(x, y, z) = c$ is a family of equipotential surfaces

(b) Show that the surfaces $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) + a^4 = c$ can form a family of equipotential surfaces and find the general form of the corresponding potential function.

(2×5=10 weightage)